Fatigue Life Estimates Using Goodman Diagrams

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The purpose of this paper is to review the proper methods by which spring manufacturers should estimate the fatigue life of a helical compression springs during the design phase. The paper will begin with a working definition of fatigue and a brief discussion of fatigue characteristics. A short history will be presented as to how fatigue test data has been evaluated historically (e.g., S-N curves, Weibull distribution, modified Goodman diagrams, etc.) Spring-specific Modified Goodman diagrams, presented in the new SMI Encyclopedia of Spring Design, will be discussed. These Modified Goodman diagrams have been sufficiently characterized to facilitate direct calculation of predicted life. These calculations will be presented along with a comparison to the results of traditional graphical methods.

An Overview of Fatigue

Fatigue, or metal fatigue, is the failure of a component as a result of cyclic stress. The failure occurs in three phases: crack initiation, crack propagation, and catastrophic overload failure. The duration of each of these three phases depends on many factors including fundamental raw material characteristics, magnitude and orientation of applied stresses, processing history, etc. Fatigue failures often result from applied stress levels significantly below those necessary to cause static failure.

Fatigue failures are typically characterized as either low-cycle (<1,000 cycles) or high-cycle (>1,000 cycles). The threshold value dividing low- and high-cycle fatigue is somewhat arbitrary, but is generally based on the raw material's behavior at the microstructural level in response to the applied stresses. Low- cycle failures typically involve significant plastic deformation. An example would be reversed 90° bending of a paperclip. Gross plastic deformation will take place on the first bend, but failure will not occur until approximately 20 cycles. Plastic deformation does play a role in high cycle fatigue; however, the plastic deformation is very localized and not necessarily discernable by a macroscopic evaluation of the component. In summary, while a valve spring designer may consider a failure at 10,000 cycles very short life, the failure can still be the result of high-cycle fatigue because the material response at the microstructural level is the same as in a 10,000,000-cycle failure under lower applied stresses.

Most metals with a body centered cubic crystal structure have a characteristic response to cyclic stresses. These materials have a threshold stress limit below which fatigue cracks will not initiate. This threshold stress value is often referred to as the endurance limit. In steels, the life associated with this behavior is generally accepted to be 2×10^6 cycles. In other words, if a given stress state does not induce a fatigue failure within the first 2×10^6 cycles, future failure of the component is considered unlikely. For spring applications, a more realistic threshold life value would be 2×10^7 cycles. Metals with a face center cubic crystal structure (e.g. aluminum, austenitic stainless steels, copper, etc.) do not typically have an endurance limit. For these materials, fatigue life continues to increase as stress levels decrease; however, a threshold limit is not typically reached below which infinite life can be expected.

The S-N Curve

The discussion above touched on the relationship between applied stress and expected life. For the designer, it is critical that this relationship be characterized so that fatigue life can be predicted. One of the early methods for characterizing this relationship is the S-N curve. 'S' stands for the cyclic stress range while 'N' represents the number of cycles to failure.

To develop the curve, a series of samples is tested to failure at various stress ranges. The resulting lives are plotted versus the corresponding stress range. The S-N curve is the locus of these data points. In more thorough testing, multiple samples are tested at each stress range. Common practice is to plot the S-N curve through the mean value at each stress range. An example is shown in Figure 1. Note that in this case, a fatigue stress below 49 kpsi will not induce failure.

One significant limitation of the S-N curve is that the resulting plot is highly dependent on the test conditions (e.g. the stress ratio $S_{min}/S_{max,}$ sample geometry, sample surface condition, and material). Using an S-N curve to predict real-world life when conditions do not match the test conditions under which the curve was developed is dubious at best. This severely limits the use of S-N curves in product design. On the other hand, the ease of construction makes the S-N curve a simple and valuable tool in making relative comparisons between materials or process variations.
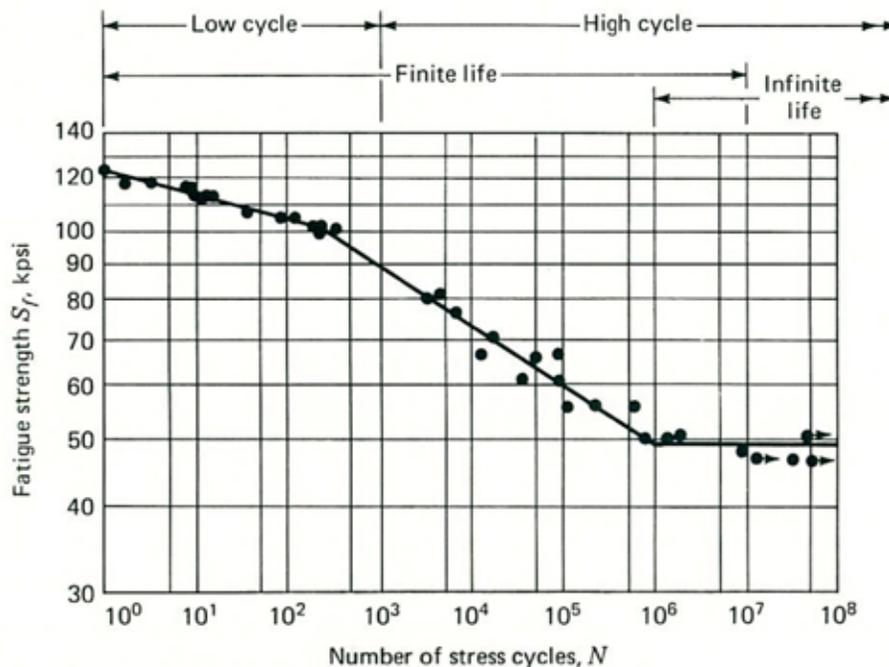

Figure 1. An S-N diagram plotted from the results of completely reversed axial fatigue tests. Material: UNS G41300 steel normalized; S_{ut}=116 kpsi; maximum S_{ut}=125 kpsi. *(Data from NACA Technical Note 3866, December 1966.)*[1]

Weibull Distribution

Individual fatigue failures are impossible to predict. Failures from a given set of test conditions are distributed randomly within a fairly predictable distribution function. The most familiar type of distribution function is the normal or Gaussian distribution. Unfortunately, fatigue failures do not usually follow this type of distribution. Consequently, proper analysis requires more sophisticated statistical techniques. A normal distribution can be fully characterized with two parameters: mean and standard deviation. A more generic method of describing distributions is through the Weibull function. A Weibull distribution can also usually be characterized by two parameters. First is the shape parameter (β). As its name implies, this parameter describes the shape of the distribution. The second parameter is the characteristic life (η). From $-\infty$ to this value, the area under the normalized probability distribution function is 0.632. In certain cases, a third parameter is necessary to identify the minimum expected life (x_0). In evaluating spring fatigue life, the third parameter is generally accepted to be zero and, therefore, will not be discussed further here.
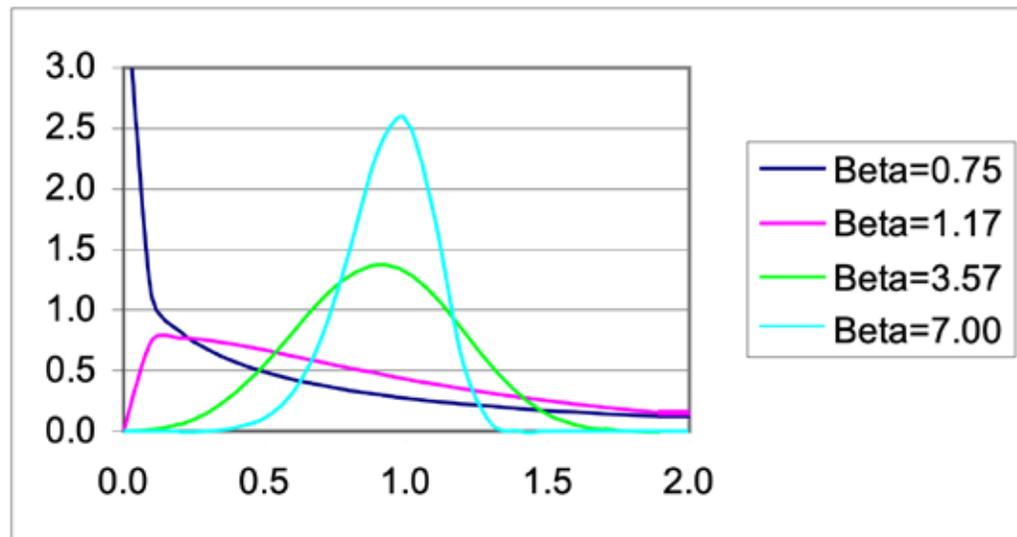

Figure 2. A plot of Weibull distributions for different shape factors. The characteristic life for all three distributions is equal to 1.0.

Figure 2 shows distribution curves for various shape parameters. It is obvious that significantly different distribution shapes can be described by the Weibull function. (Incidentally, the normal distribution curve is a special case of the Weibull distribution with a shape factor β approximately equal to 3.57.) As Figure 2 shows, the shape parameter β also provides an indication as to the "width" of the distribution. The larger the shape factor, the more condensed the data.

Obtaining the distribution parameters from a set of data can be somewhat involved. Historically, the shape parameter was determined graphically. The following steps are oversimplified and are not intended to instruct the reader in the graphic methodology. Rather, the steps are presented to give the reader an insight as to the complexity of the task.

Constructing a Weibull Plot

1. The data is ordered from shortest life to longest.

2. A regression analysis is conducted to calculate the median rank for each life value. (The regression technique may vary based on sample size, type of data, etc.) The median rank is a value between zero and one and approximately represents the fraction of the distribution expected to exist below the specific life value.

3. The double logarithm 1/(1-median rank) is plotted vs. the logarithm of the actual life data. The data points typically fall very near a straight line. Special Weibull paper has been created to ease this task.

4. A best-fit line is drawn through the data points.

5. The shape parameter is the slope of the best-fit line.

6. A horizontal line is constructed at a y value of 0.632.

7. The characteristic life is the x value at the intersection of the horizontal line and the best-fit line through the data points.

Many software packages are available which automate the calculations. The software typically also calculates the Weibull parameters rather than determine them graphically. A Weibull plot is shown in Figure 3 for failures occurring at the following intervals (61,000; 91,000; 114,000; 135,000; 155,000; 177,000; 205,000; 245,000). Note that the shape parameter and characteristic life have been calculated and are shown on the plot (β=2.411, η=168,438).
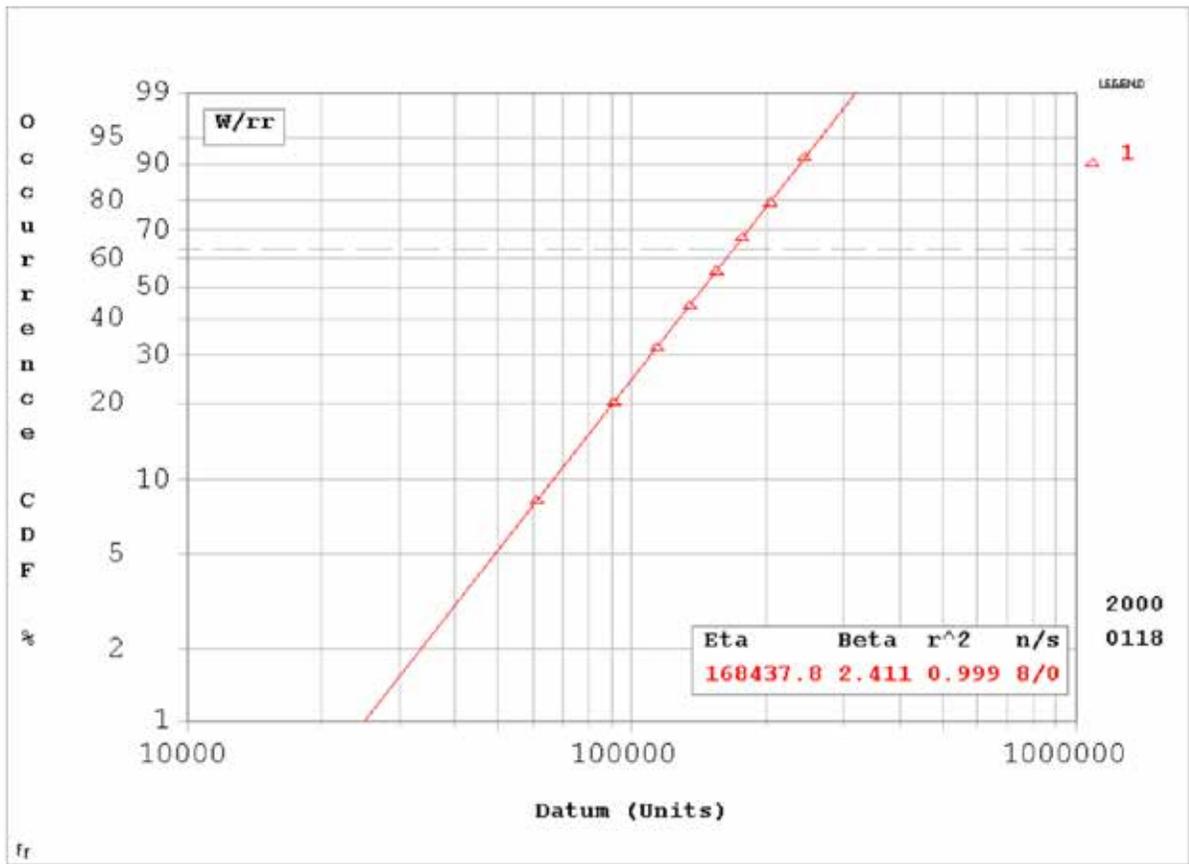
Figure 3. Example of a Weibull Plot.

Interpreting a Weibull Plot

The Weibull plot can be a very powerful tool in making life predictions based on a set of data. Before proceeding further, however, it is necessary to define some terms. When evaluating fatigue data, the Weibull plot is the unreliability (U) of the population expressed as a function of life (x), U=f(x). Life can be expressed in terms of time, cycles, etc. The unreliability at any specified life is the fraction of the population expected to fail before that life is achieved. Reliability (R) is the fraction of the population expected to survive at the specified life. The sum of the Reliability and Unreliability equals 1. In other words, the y-axis of the Weibull plot represents both U and 1-R.

A very important and often misunderstood term related to Weibull analysis is confidence (C). Think of confidence as a measure of error in determining the Weibull parameters. By definition, the best-fit line through the data on the Weibull plot represents a confidence level of 50%. In other words, if additional data were collected from the same sample population used to generate a Weibull plot, the next data point would have a 50% chance of falling to the right side of the existing 50% Confidence line. An additional interpretation is demonstrated in the following example.

Example 1.

The Weibull plot shown in Figure 3 suggests 20% unreliability at a life of 90,426 cycles at a confidence level of 50%. Assume a total of eight tests are to be conducted. The test apparatus is configured to test ten pieces simultaneously. It can be expected that in four of the tests (C=50%), more than two pieces will fail (U=20%) when testing reaches 90,426 cycles. In the remaining four tests, it can be expected that less than two pieces will fail when testing reaches 90,426 cycles.

Auxiliary curves can be constructed on the Weibull plot to represent different confidence levels. For a given confidence level, the width of the band between the C=50% line and the new curve is a function of the number of data points in the data set. In other words, the more data used to calculate the distribution, the more certainty there is in the calculated results. Confidence levels greater than 50% are shown to the left of the C=50% line while those lower than 50% are shown to the right. Typically, confidence levels below 50% are not considered in the spring industry. Figure 4 shows the 90% confidence band added to the Weibull plot in Figure 3. Similar to the scenario presented above, assume that additional data were collected from the same sample population used to generate the plot in Figure 3. The next data point would have a 90% chance of falling to the right of the existing 90% Confidence line, but only a 50% chance of falling to the right of the existing 50% Confidence line.

Example 2.

Using Figure 4, the life at which there is 90% confidence in 20% unreliability is 64,591 cycles. Again, the life at U=20%, C=50% is 90,426. In this example ten tests will be run with ten pieces in each test. The following predictions can be made:

- In five of the tests, fewer than two pieces will fail before testing reaches 90,426 cycles. (C=50%, U=20% at x=90,426)

- In the remaining five tests, more than two pieces will fail before testing reaches 90,426 cycles. (C=50%, U=20% at x=90,426)

- In nine of the tests, fewer than two pieces will fail before testing reaches 64,591 cycles. (C=90%, U=20% at x=64,591)

- In one of the tests, more than two pieces will fail before testing reaches 64,591 cycles. (C=90%, U=20% at x=64,591)

The bearing industry created a standard for succinctly stating the predicted life of a component based on the Weibull distribution of failures. Common practice has become to state the predicted life at 10% unreliability. This became known as the B10 life. Of course, the B10 life can be stated at various confidence levels. SAE standard is to state the B10 life at 50% confidence[2]. For clarification, the confidence level is often stated preceding the B10 designation. For example, a B10 life at 90% confidence of 250,000 cycles would be stated as '90B10 = 200,000.' When a confidence level is not specified, it is typically assumed to be 50%.
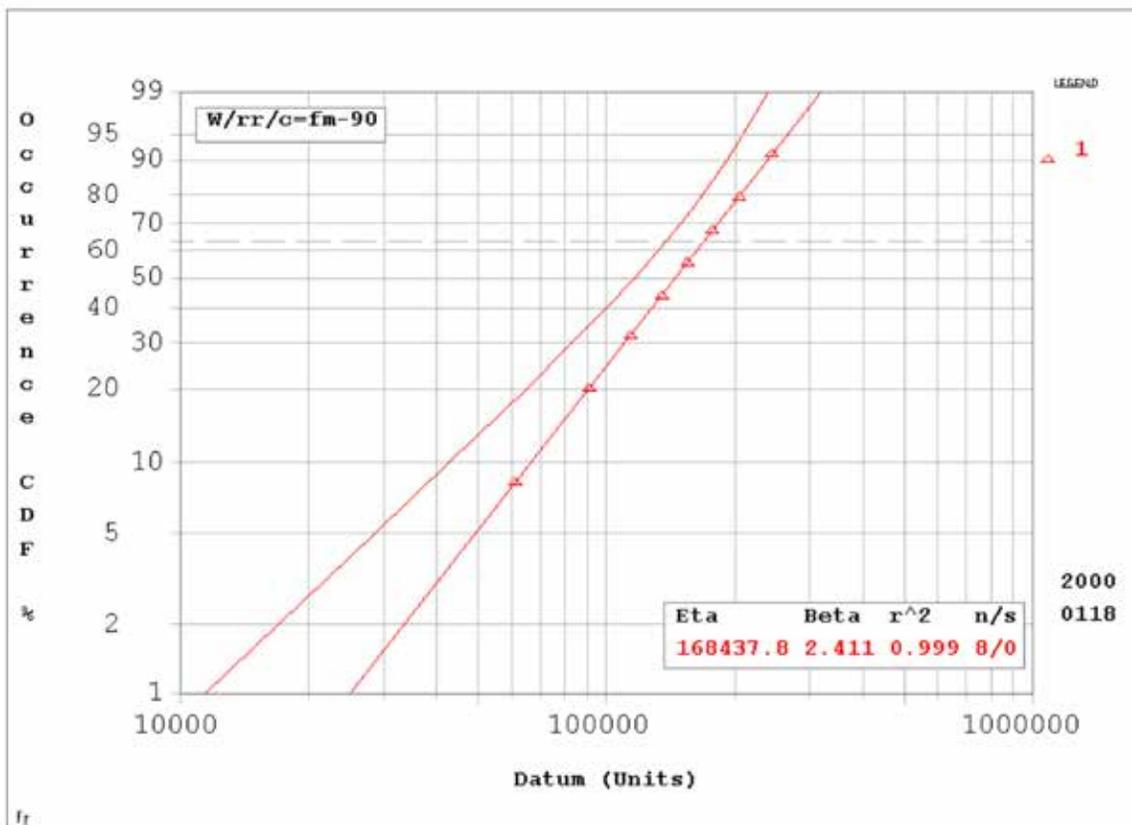

Figure 4. Weibull Plot Showing 90% Confidence Band

Modified Goodman Diagram

One of the key limitations to the S-N curve was the inability to predict life at stress ratios different from those under which the curve was developed. In predicting the life of a component, a more useful presentation of fatigue life test data is the modified Goodman Diagram. These diagrams, while still limited by specimen geometry, surface condition, and material characteristics, afford the user to predict life at any stress ratio. Typically, modified Goodman diagrams are developed for specific applications. Of course, use of the diagram is also limited to that application.

Modified Goodman diagrams are presented in a variety of formats. The most common format used in the spring industry has the minimum operating stress along the x-axis while the maximum operating stress is along the y-axis. Sufficient test data is generated to know the maximum and minimum stresses at various points that provide the same known life. Each of these points is plotted on the diagram. A line is then drawn through these points. Any combination of maximum and minimum stress that fall on the plotted line will be expected to have the known life. Points below the line will have a longer life, points above the line represent shorter life. Additional features are present indicating the material or service limits. The reader is invited to explore these details independently. A thorough discussion is presented in the SAE HS-795 Design and Application of Helical and Spiral Springs section 6[3].

A great deal of testing has been conducted to develop modified Goodman diagrams specifically for helical compression springs. It has been found that, within a select set of spring materials, normalizing the plotted data by dividing by the material's tensile strength allows the use of the same diagram for a variety of materials across their range of tensile strengths. A typical example from SAE HS-795 is presented in Figure 5. This diagram was developed using B10 test lives. As a result, life estimates based on this diagram are B10 estimates at 50% Confidence.

To use this diagram, the designer must first know the spring's application. Specifically, the minimum and maximum Wahl corrected stress must be known for the spring's operating range. These will be referred to as S_1 and S_2, respectively. (Note that the Wahl correction factor must be calculated appropriately based on spring manufacturing process, e.g., preset or not preset.) The minimum ultimate tensile strength, S_U must also be known. The stresses are normalized by dividing the operating stresses by the minimum ultimate tensile strength. The resulting numbers are referred to as the stress factors. Mathematically, $K_{S1} = S_1/S_U$ and $K_{S2} = S_2/S_U$. It is important to note that the maximum value for K_{S2} varies with product life. The point on the diagram is located corresponding to the two stress factors. The B10 life is approximated based on the nearest known life curves for the given spring process.

Example 3.

A music wire spring is under consideration which has the following dimensions.

d = .187″

D = .625″

n = 7

FL = 1.75″

In service, the spring will cycle between heights of 1.69″ and 1.45″.

Per customer requirements, the spring is to be preset and shot peened. The customer has asked for an estimated life. The customer has also asked what the anticipated life would be if shot peening were eliminated as a cost saving measure.
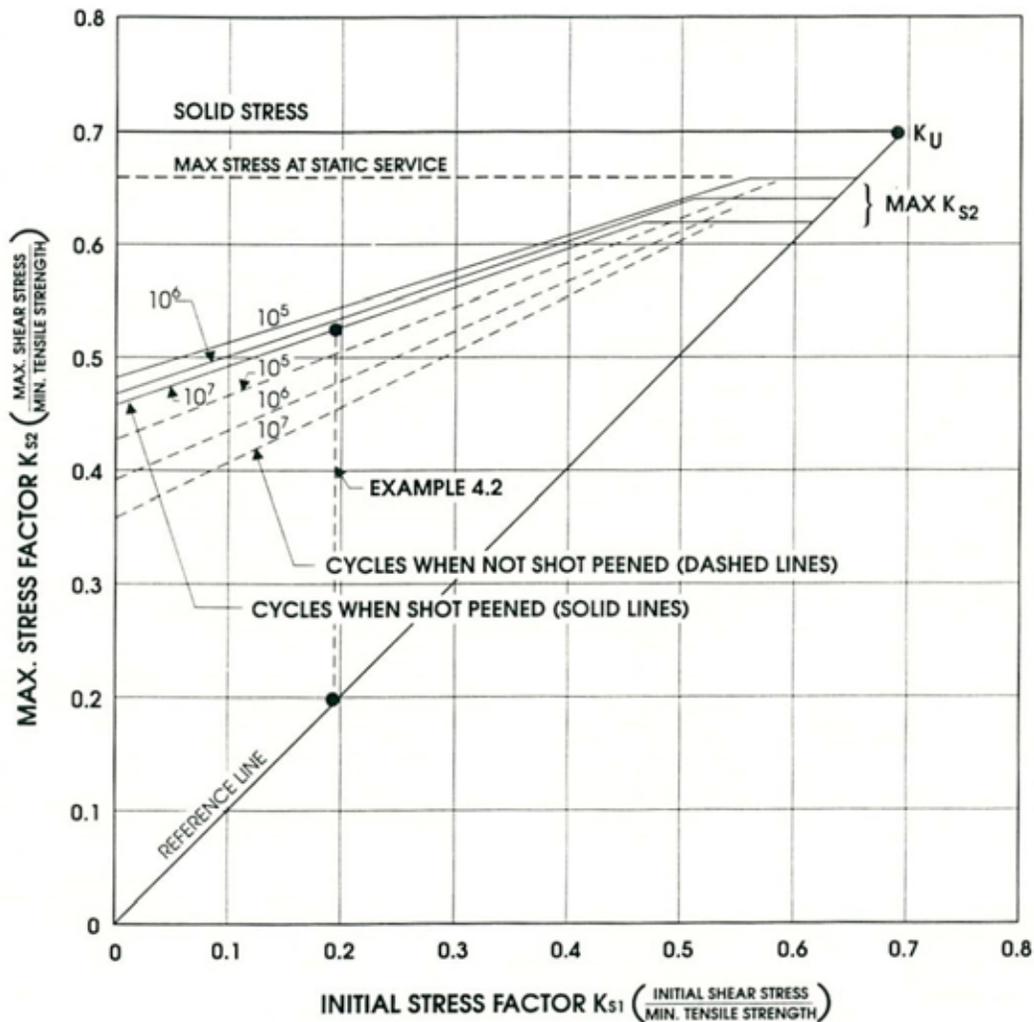

Figure 5. Fatigue strength diagram for round wire helical compression springs which are preset. All stresses are Wahl corrected with diagram representing a B-10 fatigue life. Diagram applicable to moderately preset springs and the following materials: Music Steel Spring Wire and Springs – SAE J178; Hard Drawn Carbon Valve Spring Quality Wire and Springs – SAE J172; Oil Tempered Carbon Valve Spring Quality Wire and Springs – SAE J351; Oil Tempered Chromium-Vanadium Valve Spring Quality Wire and Springs – SAE J132.[4]

Start by calculating the stress at each load height.

- $k = Gd^4/(8D^3N_a) = 1440$ lb/in

- $P_1 = 86.4$ lb, $P_2 = 432.0$ lb

- $C = D/d = 3.342$

- $K_{W2} = 1 + 0.5/C = 1.15$

- $S = 8PDK_W/(\pi d^3)$

- $S_1 = 24{,}175$ psi, $S_2 = 120{,}873$ psi

- From tabulated data, the minimum ultimate tensile strength for music wire is 242,000 psi.

- $K_{S1} = 24{,}175 / 242{,}000 = 0.10$

- $K_{S2} = 120{,}873 / 242{,}000 = 0.50$

- Referring to Figure 5 above, follow the line up at $K_{S1} = 0.10$ until it reaches the value for $K_{S2} = 0.50$. This intersection is slightly below the 10^6-cycle line for shot peened springs. Therefore, this spring would be expected to have a B10 life greater than 10^6 cycles but significantly less than 10^7 cycles. A rough estimate would be 2×10^6 cycles.

- Comparing the plotted point to the curves for unpeened springs, it is obvious that the life will be well below 10^5 cycles. Extrapolating from the curves, unpeened B10 life is estimated to be approximately 10^4 cycles. Therefore, eliminating shot peening would be responsible for a 100-fold reduction in life.

Based on the graphical interpretation, the accuracy of these estimates is poor. The magnitude of error could easily be a factor of 3. This approach is best used for predicting life in terms of the order of magnitude.

Please note that Figure 5 above refers specifically to cold-wound preset springs. Additional modified Goodman diagrams are presented in SAE HS-795 for springs that are cold-wound and not preset as well as hot wound and preset springs. It is imperative that the modified Goodman diagram be selected which is representative of the actual manufacturing process. The careful reader will note that Chrome-Silicon and Chrome-Silicon-Vanadium alloys are not specifically listed as materials to which this diagram applies.

Common practice, based on limited historical data, is to consider the SAE cold-wound diagrams applicable to these materials as well. Similarly, no distinction is typically made between hot- and cold-set parts. In both cases, the springs are simply considered preset. In the event that significantly different materials are introduced (i.e., titanium, stainless steels, etc.), it may be necessary to perform structured testing with the intent of either confirming the applicability of the given modified Goodman diagrams or creating new material-specific modified Goodman diagrams.

One significant change in the new SMI Encyclopedia of Spring Design is in the Modified Goodman Diagrams themselves. The old diagrams claimed to be applicable to a very broad variety of material types, including cold drawn and oil tempered steels as well as stainless steels. Given that austenitic stainless steels do not have an endurance limit as would be expected in either cold drawn or oil tempered steel, the relationship between the lines of a Goodman diagram, particularly in the 10^5 to 10^7 cycle range would be significantly different. Therefore, these materials could not share a normalized Modified Goodman Diagram. At the time of developing the new Encyclopedia of Spring Design, the best available Modified Goodman Diagrams with a clear research history came from SAE HS-795. The scope of application of these diagrams is more limited; however, it is believed that the improved accuracy outweighs limitations.

Calculation of Predicted Life

A great deal of effort has been invested in mathematically characterizing the modified Goodman diagrams. As a result, fatigue life estimates can be calculated directly without referring to the modified Goodman diagram.[5] The following list of parameters will be used in the calculations.

- C_E, C_S, M, and Y – Constants (process-specific)
- K_E – Endurance Factor
- K_{S1} – Initial Stress Factor
- K_{S2} – Maximum Stress Factor
- K_U – Ultimate Strength Factor (process-specific, approximately equal to solid stress divided by minimum ultimate tensile strength)
- n – Life Cycles
- S_1 – Initial Stress
- S_2 – Maximum Stress
- S_U – Minimum Ultimate Tensile Strength

First, the initial stress factor, the maximum stress factor, and the endurance factor must be calculated.

- $K_{S1} = S_1 / S_U$
- $K_{S2} = S_2 / S_U$
- $K_E = \dfrac{K_U(K_{S2} - K_{S1})}{2K_U - (K_{S2} + K_{S1})}$

The endurance factor is related to life through the following relationship which includes process-specific parameters.

- $K_E = C_E n^Y$

This relationship can be re-ordered to a more useful format.

- $n = \exp[(1/y)\ln(K_E/C_E)]$

As a final confirmation, ensure that K_{S2} has not exceeded the maximum value for the calculated life.

- $K_{S2MAX} = C_S n^M$

The process-specific parameters are provided in Table 1[6]. Again, it is imperative that that the parameters used match the manufacturing process for the spring under consideration.

Table 1. Modified Goodman Diagram Calculation Parameters

Process			Parameters				
Coiling	Preset	Shot Peened	K_U	C_S	M	C_E	Y
Cold	No	No	0.5600	0.5546	-0.0090	0.6620	-0.0622
		Yes				0.5021	-0.0206
	Yes	No	0.7000	0.7757	-0.0139	0.5758	-0.0537
		Yes				0.4579	-0.0180
Hot	Yes	Yes	0.740	0.830	-0.0215	1.808	-0.130

The parameters in Table 1 correlate fairly well with the graphic method when the calculated life falls between 5×10^5 and 10^7 cycles. Outside of this range, the predictions become highly unreliable, particularly when predicting shorter lives. When dealing with very large or very small calculated lives, it is recommended that the calculations only be used for relative comparison of results rather than a prediction of real-world life.

Example 4.

For the spring in example 3, calculate the B10 life in both the peened and unpeened conditions.

From above,

- $S_1 = 24{,}175$ psi

- $S_2 = 120{,}873$ psi

- $S_U = 242{,}000$ psi

- $K_{S1} = S_1 / S_U = 0.10$

- $K_{S2} = S_2 / S_U = 0.50$

- $K_U = 0.7000$ (cold wound, preset)

- $K_E = \dfrac{K_U(K_{S2} - K_{S1})}{2K_U - (K_{S2} + K_{S1})} = \dfrac{0.7000(0.50\text{-}0.10)}{2(0.70000)\text{-}(0.50+0.10)} = 0.35$

For the *shot peened* condition,

- $C_E = 0.4579$ (cold wound, preset, shot peened)
- $Y = -0.0180$ (cold wound, preset, shot peened)

- $n = \exp[(1/Y)1n(K_E/C_E)] = \exp[(1/\text{-}0.0180)\ln(0.35/0.4579)] = \underline{3{,}044{,}231 \text{ cycles}}$

Finally confirm that K_{S2} does not exceed the limiting value for this life.

- $CS = 0.7757$
- $M = -0.0139$

- $K_{S2MAX} = C_S n^M = (0.7757)(3{,}044{,}231)^{-0.0139} = 0.63$
- $K_{S2} < K_{S2MAX}$ Therefore, the estimate is valid.

For the *unpeened* condition substitute the following values.

- $C_E = 0.5758$ (cold wound, preset, unpeened)

- $Y = -0.0537$ (cold wound, preset, unpeened)

- $n = \exp[(1/Y)\ln(K_E/C_E)] = \exp[(1/-0.0537)\ln(0.35/0.5758)] = \underline{10,620 \text{ cycles}}$

- $K_{S2MAX} = C_S n^M = (0.7757)(10,620)-0.0139 = 0.68$

- $K_{S2} < K_{S2MAX}$ Therefore, the estimate is valid.

The calculated life for the peened spring is 50% greater than that estimated in Example 3. As stated above, this is within the expected error of the graphic estimates. Comparing the results for the unpeened spring shows surprising agreement.

References

1. Shigley, J.E. and Mitchell, L.D., Mechanical Engineering Design, Fourth Edition, McGraw-Hill, New York, 1983, p. 273.

2. Spring Design Manual, Second Edition, Society of Automotive Engineers, Warrendale, 1996, p. 90.

3. ibid, p. 196.

4. ibid, p. 193.

5. ibid, p. 142.

6. ibid, p. 195.